Amplitudes for Higgs Bosons plus Four Partons

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In this talk we consider amplitudes for processes involving a Higgs boson, either scalar or pseudoscalar, plus four light partons. These amplitudes are relevant to the production of a Higgs boson plus two jets in hadronic collisions. They are also relevant to calculating the transverse momentum spectrum for Higgs bosons at next-to-leading order in the strong coupling. We work in the limit that the top quark is much heavier than the Higgs bosons and use effective Lagrangians for the interactions of gluons with the Higgs bosons. We present the amplitudes involving a Higgs boson and: 1) four gluons; 2) two quarks and two gluons; and 3) four quarks. We show that the pseudoscalar amplitudes are nearly identical to those for the scalar case, the only differences being the overall size and the relative signs between terms.

I. INTRODUCTION

Extensions of the Standard Model with enlarged Higgs sectors have richer particle content than the minimal Standard Model; in general, neutral pseudoscalars (with respect to their fermion couplings) and charged scalars as well as extra neutral scalars are present [1]. In this paper we will study the amplitudes involving a neutral Higgs boson, scalar (H) or pseudoscalar (A), and four light partons. This talk summarizes published work in Refs. [2–4].

We will present helicity amplitudes for the following processes:

$$\begin{aligned} &H(A) \to gggg \\ &H(A) \to q\bar{q}gg \\ &H(A) \to q\bar{q}q^{(\prime)}\bar{q}^{(\prime)}. \end{aligned} \tag{1}$$

These amplitudes are a part of the calculation of the next-to-leading order transverse momentum distribution for Higgs bosons. They can be combined with the virtual corrections to the production of a Higgs boson plus one jet to make the total cross section. The production of a Higgs boson plus two jets has interest in its own right because it provides information about the environment in which Higgs bosons are produced, allowing one to address the question of how often there are extra jets in events with Higgs bosons. Lastly, these amplitudes would also be a part of the calculation of the next-to-next-to-leading order cross section for Higgs boson production.

II. EFFECTIVE LAGRANGIANS

We will employ the approximation that the Higgs boson is much lighter than the top quark: $M \ll m_t$. This greatly simplifies the calculation and is reasonably accurate even when $M \sim m_t$. Furthermore, the shape of the Higgs-boson $p_{\rm T}$ distribution at low $p_{\rm T}$ is independent of m_t and so the approximate results give some information about heavy Higgs bosons as well. We use the following effective Lagrangians [5,6] to couple the Higgs bosons to gluons:

$$\begin{aligned} \mathcal{L}_{\rm H} &= -\frac{1}{4} g_{\rm H} G^a_{\mu\nu} G^a_{\mu\nu} H, \\ \mathcal{L}_{\rm A} &= g_{\rm A} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu} A, \end{aligned} \tag{2}$$

where $G^a_{\mu\nu}$ is the field strength of the SU(3) color gluon field, $\tilde{G}^a_{\mu\nu}$ is its dual, $\tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma}$, and H, A are the Higgs fields. We choose the couplings between the top quark and the Higgs scalar and pseudoscalar to be m_t/v

and $m_t\gamma_5/v$, respectively, where v is the vacuum expectation value parameter, $v^2 = (G_F\sqrt{2})^{-1} = (246\text{ GeV})^2$. The effective couplings to gluons are then given by $g_{\rm H} = \alpha_s/(3\pi v)$ and $g_{\rm A} = \alpha_s/(8\pi v)$.

The effective Lagrangians generate vertices involving the Higgs bosons and two, three, or four gluons. We assign outgoing gluon momenta and spin indices $p_1^\mu, p_2^\nu, p_3^\rho, p_4^\sigma$ and color indices a, b, c, d. Labeling the vertex involving a Higgs scalar and n gluons as $V_n^{\rm H}$ we have

$$\begin{aligned}
V_2^{\rm H} &= ig_{\rm H}\delta^{ab}(g^{\mu\nu}p_1\cdot p_2 - p_1^\nu p_2^\mu),\\
V_3^{\rm H} &= -gg_{\rm H}f^{abc}\left[(p_1-p_2)^\rho g^{\mu\nu} + (p_2-p_3)^\mu g^{\nu\rho} + (p_3-p_1)^\nu g^{\rho\mu}\right],\\
V_4^{\rm H} &= -ig^2 g_{\rm H}\left[f_{abe}f_{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f_{ace}f_{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + f_{ade}f_{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma}).\right]
\end{aligned} \tag{3}$$

In similar notation, the vertices for the Higgs pseudoscalar are

$$\begin{aligned}
V_2^{\rm A} &= -ig_{\rm A}\delta^{ab}\epsilon^{\mu\nu\rho\sigma}p_{1\rho}p_{2\sigma},\\
V_3^{\rm A} &= -gg_{\rm A}f^{abc}\epsilon^{\mu\nu\rho\sigma}(p_1+p_2+p_3)_\sigma.
\end{aligned} \tag{4}$$

The four-gluon pseudoscalar vertex vanishes as it is proportional to the completely antisymmetric combination of structure constants:

$$f^{abe}f^{cde} - f^{ace}f^{bde} + f^{ade}f^{bce} = -2\mathrm{tr}\{[T^a,T^b][T^c,T^d] - [T^a,T^c][T^b,T^d] + [T^a,T^d][T^b,T^c]\} = 0, \tag{5}$$

where the T^i are the SU(3) generators.

III. SPINOR-PRODUCT FORMALISM

We will compute helicity amplitudes using a spinor-product formalism [7,8]. We refer the reader to Ref. [3] for details of our implementation. We will use the convention that all the particles are outgoing. The momenta of the massless particles are labeled $p_1,\ p_2,\ p_3,\ p_4$ and the momentum of the Higgs boson is labeled p. We will use the shorthand notations $\langle ij\rangle$ to denote a left-handed spinor with momentum p_i contracted on the left with a right-handed spinor with momentum p_j and $[ij]$ to represent such a contraction with the right-handed spinor on the left. We also define the invariant masses $S_{ij} = (p_i+p_j)^2$ and $S_{ijk} = (p_i+p_j+p_k)^2$.

Amplitudes for processes involving the Higgs pseudoscalar contain expressions of the form $\epsilon_{\mu\nu\rho\sigma}w^\mu x^\nu y^\rho z^\sigma$ where w, x, y, z are momenta, polarization vectors, or fermion currents. These contractions can be written in terms of spinor products through the following procedure [4]. We first write

$$\epsilon_{\mu\nu\rho\sigma}w^\mu x^\nu y^\rho z^\sigma = \frac{1}{4i}\mathrm{tr}\{\not{w}\not{x}\not{y}\not{z}\gamma_5\} = \frac{1}{4i}\mathrm{tr}\{\not{w}\not{x}\not{y}\not{z}(P_+ - P_-)\}, \tag{6}$$

where the projection operators are $P_\pm = (1\pm\gamma_5)/2$. Each slashed vector can be written in terms of outer products of spinors:

$$\not{w} = w_+|w_1+\rangle\langle w_2+| + w_-|w_3-\rangle\langle w_4-|. \tag{7}$$

Inserting Eq. (7) into Eq. (6) and expressing the trace in terms of matrix multiplication, we have

$$\epsilon_{\mu\nu\rho\sigma}w^\mu x^\nu y^\rho z^\sigma = \frac{1}{4i}(w_+\langle w_2+|\not{x}\not{y}\not{z}|w_1+\rangle - w_-\langle w_4-|\not{x}\not{y}\not{z}|w_3-\rangle), \tag{8}$$

which reduces to spinor products upon application of Eq. (7) to $\not{x}$, $\not{y}$, and $\not{z}$.

IV. LIMITS

One of the virtues of computing helicity amplitudes in the spinor product formalism is that their limiting behavior is easily made manifest. We know on general grounds that in the limit that an external parton becomes soft or two particles become collinear the Higgs boson plus four parton amplitudes must factorize into a sum of eikonal factors times amplitudes for Higgs bosons plus three partons [9]. Additionally, we know that the amplitudes for Higgs scalars going to three partons are identical, up to couplings and phases, to those for Higgs pseudoscalars [6,4].

The amplitudes also have specific limits when the four-momentum of the Higgs boson goes to zero. The scalar amplitudes become proportional to the pure QCD amplitudes for four partons. The pseudoscalar amplitudes, on the other hand, must vanish. This behavior is due to the coupling of the pseudoscalar field to $G\tilde{G}$, which is a total derivative. Inspection of the vertices involving the pseudoscalar, Eq. (4), shows that they are proportional to the pseudoscalar momentum: $V_2^{\mathrm{A}} \sim \epsilon^{\mu\nu\rho\sigma} k_{1\rho} p_\sigma$ and $V_3^{\mathrm{A}} \sim \epsilon^{\mu\nu\rho\sigma} p_\sigma$.

V. RESULTS

The four-gluon amplitudes are written in the dual-color decomposition [9–11]. The scattering amplitude for a Higgs boson and four gluons with external momenta p_1,...,p_4, colors a_1,...,a_4, and helicities λ_1,...,λ_4 is

$$\mathcal{M}_{(\mathrm{H,A})} = 2 g_{(\mathrm{H,A})} g^2 \sum_{\mathrm{perms}} \mathrm{tr}(T^{a_1} ... T^{a_4}) m(p_1, \epsilon_1; ...; p_4, \epsilon_4), \tag{9}$$

where the sum is over the non-cyclic permutions of the momenta. For the helicity choices $++++$ and $-+++$ the subamplitudes for Higgs pseudoscalars are identical to their scalar counterparts and can be obtained from the following [2–4]:

$$m(1^+, 2^+, 3^+, 4^+) = \frac{M^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \tag{10}$$

$$\begin{aligned} m(1^-, 2^+, 3^+, 4^+) = &-\frac{\langle 1-|\not{p}|3-\rangle^2 [24]^2}{S_{124} S_{12} S_{14}} - \frac{\langle 1-|\not{p}|4-\rangle^2 [23]^2}{S_{123} S_{12} S_{23}} - \frac{\langle 1-|\not{p}|2-\rangle^2 [34]^2}{S_{134} S_{14} S_{34}} \\ &+ \frac{[24]}{[12]\langle 23\rangle\langle 34\rangle [41]} \left\{ S_{23} \frac{\langle 1-|\not{p}|2-\rangle}{\langle 41\rangle} + S_{34} \frac{\langle 1-|\not{p}|4-\rangle}{\langle 12\rangle} - [24] S_{234} \right\}. \end{aligned} \tag{11}$$

The subamplitudes for the helicity choice $--++$ differ between scalar and pseudoscalar by a relative sign between terms. All the subamplitudes for this case can be obtained from

$$m_{(\mathrm{H,A})}(1^-, 2^-, 3^+, 4^+) = -\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \mp \frac{[34]^4}{[12][23][34][41]}, \tag{12}$$

where the upper sign goes with the H and the lower with the A.

As was the case for the four-gluon amplitudes, the calculation of the $q\bar{q}gg$ amplitudes can be simplified by a judicious choice of color decomposition [12,10]. We assign momentum labels 1,2,3,4 and color labels i, j, a, b to the quark, anti-quark and two gluons, respectively. The amplitude is then written

$$\mathcal{M}_{(\mathrm{H,A})} = -i g^2 g_{(\mathrm{H,A})} \sum_{\mathrm{perms}} (T^a T^b)_{ij} m(p_3, \epsilon_3; p_4, \epsilon_4), \tag{13}$$

where the sum runs over the two gluon permutations. We label the helicity amplitudes by the helicity of the quark, anti-quark and the two gluons (in that order). The quark and anti-quark always have opposite helicities. In the case where the gluons have identical helicities we find that the scalar and pseudoscalar amplitudes are identical [3,4]:

$$m^{+-++}(3,4) = \frac{\langle 2-|\not{p}|3-\rangle^2}{S_{124}} \frac{[14]}{\langle 24\rangle} \left(\frac{1}{S_{12}} + \frac{1}{S_{14}} \right) - \frac{\langle 2-|\not{p}|4-\rangle^2}{S_{123} S_{12}} \frac{[13]}{\langle 23\rangle} + \frac{\langle 2-|\not{p}|1-\rangle^2}{[12]\langle 23\rangle\langle 24\rangle\langle 34\rangle}. \tag{14}$$

To get the subamplitude with the other ordering, $m^{+-++}(4,3)$, exchange $p_3 \leftrightarrow p_4$ in this expression. When the two gluons have opposite helicities the scalar and pseudoscalar amplitudes differ by a relative sign between terms:

$$m_{(\mathrm{H,A})}^{+-+-}(3,4) = \frac{[13]^3}{[12][14][34]} \mp \frac{\langle 24\rangle^3}{\langle 12\rangle\langle 23\rangle\langle 34\rangle} \tag{15}$$

$$m_{(\mathrm{H,A})}^{+-+-}(4,3) = -\frac{[13]^2[23]}{[12][24][34]} \pm \frac{\langle 14\rangle\langle 24\rangle^2}{\langle 12\rangle\langle 13\rangle\langle 34\rangle}, \tag{16}$$

where the upper signs go with the H and the lower with the A. The rest of the subamplitudes can be obtained by parity, charge conjugation and Bose symmetry transformations.

In the case of the four-quark amplitude with two different quark pairs the helicity of each quark must be opposite to its anti-quark partner. We choose momentum labels 1,2,3,4 and color labels i, j, k, l for the first quark, first anti-quark, second quark and second anti-quark respectively and label the helicities in that order. All the helicity amplitudes (including those for identical quark pairs) can be obtained from [3,4]

$$\mathcal{M}_{(\mathrm{H,A})}^{+-+-} = i g_{(\mathrm{H,A})} g^2 T_{ij}^a T_{kl}^a \left(\frac{\langle 24\rangle^2}{\langle 12\rangle\langle 34\rangle} \pm \frac{[13]^2}{[12][34]} \right), \tag{17}$$

where the upper sign is for the H and the lower for the A.

VI. CONCLUSION

The following pattern emerges from the amplitudes presented in the previous section: those amplitudes which violate helicity are identical for the scalar and pseudoscalar, modulo the different couplings; those amplitudes which conserve helicity differ for the scalar and pseudoscalar by a relative sign between two terms. This allows the soft and collinear limits to be the same for the scalar and pseudoscalar, modulo phases.

We see that the helicity-violating amplitudes vanish in the limit that the four-momentum of the Higgs boson goes to zero. This is consistent with the general result for the Higgs scalar: the scalar amplitudes reduce to pure QCD amplitudes in this limit and pure QCD conserves helicity. It is also consistent with the general result for the pseudoscalar which is that all the amplitudes must vanish when the pseudoscalar momentum goes to zero. Furthermore, we see that the helicity-conserving amplitudes for the pseudoscalar also vanish in this limit. The two terms in each expression cancel (whereas for the scalar case they add).

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